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BRST analysis of the gauged SU(2) WZW model and Darboux's transformations [1]

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Abstract

The four dimensional SU(2) WZW model coupled to elecromagnetism is treated as a constraint system in the context of the BFV approach. We show that the Darboux's transformations which are used to diagonalize the canonical one-form in the Faddeev-Jackiw formalism, transform the fields of the model into BRST invariant ones. The same analysis is also carried out in the case of spinor electrodynamics.

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1 Introduction

It is interesting to investigate the relation between the Batalin-Fradkin-Vilkovisky (BFV) quantization [1] scheme and the Faddeev-Jackiw approach [2] to constrained systems.In the first case the phase space of the theory is extended by introducing a ghost field for every constraint, while in the second case the phase space is reduced by iteratively solving the constraints and performing Darboux's transformations, until we end up with an unconstraint and canonical Lagrangian. In the first case the gauge fixing can be done in an arbitrary way by virtue of the Fradkin-Vilkovisky theorem while in the second case there is no need for gauge fixing and we proceed directly to field quantization.

Starting with the BFV formalism one can write the BFV action as sum of two terms. The first term is the uncanonical one that we would obtain with the Faddeev-Jackiw method after having solved each one of the constraints and the second term is a BRST exact one. We can see this in the case of spinor electrodynamics whose Lagrangian density is given by

$$\mathcal{L} = -\boldsymbol{\pi} \cdot \dot{\mathbf{A}} + \mathrm{i}\psi^\dagger \dot{\psi} - \mathrm{H}_0 + \mathrm{A}_0(\nabla \cdot \boldsymbol{\pi} - \rho) \tag{1}$$

where

$$H_0 = \frac{1}{2}(\boldsymbol{\pi}^2 + \mathbf{B}^2) - \psi^\dagger \boldsymbol{\alpha} \cdot (\mathrm{i}\nabla + \mathrm{e}\mathbf{A})\psi + \mathrm{m}\psi^\dagger \gamma_0 \psi$$

See Appendix for notation.

The corresponding BFV action is given by

$$S_{BFV} = \int d^4x[-\boldsymbol{\pi} \cdot \dot{\mathbf{A}} + \pi_0 \dot{\mathrm{A}}_0 + \mathrm{i}\psi^\dagger \dot{\psi} + \dot{\mathrm{C}}\mathcal{P} + \dot{\bar{\mathrm{C}}}\bar{\mathcal{P}} - \mathrm{H}_0] + \int \mathrm{dt}[\Psi, \mathrm{Q}] \tag{2}$$

The scalar potential A_0 is promoted to a full dynamical variable and its conjugate momentum π_0 has to vanish. We have also introduced the canonical pair $(C, \mathcal{P})$ of a ghost field and its conjugate momentum, corresponding to the constraint $G_1 = \rho - \nabla \cdot \boldsymbol{\pi}$, and the canonical pair $(\bar{C}, \bar{\mathcal{P}})$ of an antighost field and its canonical momentum, corresponding to the constraint $G_2 = \pi_0$. Ψ is the gauge fermion, and Q is the BRST charge. The two constraints are first class. The expression for the BRST charge is given by

$$Q = \int d^3x[C(\rho - \nabla \cdot \boldsymbol{\pi}) + i\bar{\mathcal{P}}\pi_0] \tag{3}$$

and the BRST transformations of the fields are given by

$$
\begin{aligned}
s\mathbf{A} = -\nabla \mathrm{C} \quad &, \qquad \mathrm{sC} = 0 \quad , \\
s\mathcal{P} = \nabla \cdot \boldsymbol{\pi} - \rho \quad &, \qquad s\boldsymbol{\pi} = 0 \quad , \\
sA_0 = i\bar{\mathcal{P}} \quad &, \qquad s\bar{\mathcal{P}} = 0 \quad , \\
s\bar{C} = -i\pi_0 \quad &, \qquad s\pi_0 = 0 \quad , \\
s\psi = ieC\psi \quad &, \qquad s\psi^\dagger = -ieC\psi^\dagger \quad .
\end{aligned}
\tag{4}
$$

One can easily see that the canonical Hamiltonian $\int d^3xH_0$, and the BFV action S_{BFV} are BRST invariant.

We decompose $\mathbf{A}$ and $\boldsymbol{\pi}$ into transverse and longitudinal components

$$
\mathbf{A}^{\mathrm{T}} = \mathbf{A} - \nabla \mathrm{A}^{\mathrm{L}'} \quad , \qquad \mathbf{A}^{\mathrm{L}} = \nabla \mathrm{A}^{\mathrm{L}'} \quad , \qquad \mathrm{A}^{\mathrm{L}'} = \frac{1}{\nabla^2}(\nabla \cdot \mathbf{A}) \quad ,
$$

$$
\boldsymbol{\pi}^T = \boldsymbol{\pi} - \frac{\nabla}{\nabla^2}\pi^{L'} \quad , \qquad \boldsymbol{\pi}^L = \frac{\nabla}{\nabla^2}\pi^{L'} \quad , \qquad \pi^{L'} = \nabla \cdot \boldsymbol{\pi} \quad .
$$

Next we use the relations (4) to solve for C , $\bar{\mathcal{P}}$, $\boldsymbol{\pi}^L$, π_0

$$
C = -sA^{L'} \quad , \qquad \bar{\mathcal{P}} = -isA_0 \quad , \qquad \boldsymbol{\pi}^L = \frac{\nabla}{\nabla^2}(s\mathcal{P} + \rho) \quad , \qquad \pi_0 = is\bar{C} \quad ,
$$

and we substitute into (2). The resulting expression for S_{BFV} consists of an uncanonical part and a BRST exact one. Then we perform the following Darboux's transformations [2]

$$
\psi \to \exp(ieA^{L'})\psi \quad , \qquad \psi^\dagger \to \exp(-\mathrm{ieA}^{\mathrm{L}'})\psi^\dagger \quad ,
$$

that diagonalize the uncanonical part. We have then

$$
\begin{aligned}
S_{BFV} \to \int d^4x[&-\boldsymbol{\pi}^T \cdot \dot{\mathbf{A}}^{\mathrm{T}} + \mathrm{i}\psi^\dagger\dot{\psi} - \mathrm{H_C} \\
&+ s[i\bar{C}\dot{A}_0 + \mathcal{P}\dot{A}^{L'} + \frac{1}{2}(s\mathcal{P})\frac{1}{\nabla^2}\mathcal{P} + \mathcal{P}\frac{1}{\nabla^2}\rho]] + \int dt[\Psi, Q]
\end{aligned}
\tag{5}
$$

where

$$
H_C = \frac{1}{2}[(\boldsymbol{\pi}^{\mathrm{T}})^2 + \mathbf{B}^2 - \rho\frac{1}{\nabla^2}\rho] - \psi^\dagger\boldsymbol{\alpha} \cdot (\mathrm{i}\nabla + \mathrm{e}\mathbf{A}^{\mathrm{T}})\psi + \mathrm{m}\psi^\dagger\gamma_0\psi
$$

is the Coulomb gauge Hamiltonian. Now we make the following choice for the gauge fermion

$$\Psi = -\int d^3x[i\bar{C}\dot{A}_0 + \mathcal{P}\dot{A}^{L'} + \frac{1}{2}(s\mathcal{P})\frac{1}{\nabla^2}\mathcal{P} + \mathcal{P}\frac{1}{\nabla^2}\rho] \tag{6}$$

We end up with a canonical unconstrained expression for the effective action with the longitudinal part of the vector potential cancelled out. The same expression for the action would be obtained if we used the original expression for the Lagrangian density (1) without extending the phase space of the system, but by merely solving the constraint $\nabla \cdot \boldsymbol{\pi} - \rho = 0$ for $\boldsymbol{\pi}^L$ and rediagonalizing the resulting expression for the Lagrangian density using the same Darboux's transformations. This is actually the Faddeev-Jackiw procedure. It is interesting to note that the Darboux transformed ψ and $\psi^\dagger$ fields are BRST closed and σ closed (physical) where σ is the contracting homotopy operator [3]. This can be seen if we perform the previously mentioned Darboux's transformations in (4). The transverse part of the vector potential $\mathbf{A}^{\mathrm{T}}$ which is also present in the expression for the gauge fixed action is also BRST closed and σ closed.

2 The $U_{EM}(1)$ gauged SU(2) WZW model

The $U_{EM}(1)$ gauged 4-dimensional SU(2) WZW model [4, 5] is a phenomenological model which has the symmetries of electromagnetism and those related to QCD without any extra ones. It describes the electromagnetic interactions of pions including those related to the axial anomaly.

In [6] this model was treated as a constrained system in the context of the Faddeev-Jackiw formalism. We expanded the effective action into series of powers in the pion fields $\theta_a \ \ , \ \ a = 1, 2, 3$ and we kept up to second and next up to third order terms. The Lagrangian density in the first case is given by

$$\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L}_{EM} + \mathcal{L}_{\sigma}^{(2)} + \mathcal{L}_{WZW}^{(2)} + O(\theta^3) \ \ , \qquad (7)\\
\mathcal{L}_{EM} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ \ , \\
\mathcal{L}_{\sigma}^{(2)} &= \frac{1}{2}\partial_\mu\theta_a\partial^\mu\theta_a + eA^\mu(\theta_2\partial_\mu\theta_1 - \theta_1\partial_\mu\theta_2) + \frac{e^2}{2}A_\mu A^\mu(\theta_1^2 + \theta_2^2) \ \ , \\
\mathcal{L}_{WZW}^{(2)} &= -\frac{N_c e^2}{12\pi^2 f_\pi}\epsilon^{\mu\nu\alpha\beta}A_\mu(\partial_\nu A_\alpha)\partial_\beta\theta_3 \ \ .
\end{aligned}$$

It can be written as an expression first order in time derivatives as follows

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\boldsymbol{\pi}\cdot\dot{\mathbf{A}} + \mathrm{p}_a\dot{\theta_a} - \mathrm{H}^{(2)} - \mathrm{A}_0(\rho^{(2)} - \nabla\cdot\boldsymbol{\pi}) + \mathrm{O}(\theta^3) \quad , \qquad (8)\\
H_0^{(2)} &= \frac{1}{2}[\boldsymbol{\pi}^2 + \mathbf{B}^2 + (\nabla\theta_a)^2 + \mathrm{p}_a^2] + \mathrm{e}\mathbf{A}\cdot(\theta_1\nabla\theta_2 - \theta_2\nabla\theta_1) + \frac{\mathrm{e}^2}{2}\mathbf{A}^2(\theta_1^2+\theta_2^2)\\
&\quad - \frac{N_c e^2}{6\pi^2 f_\pi}(\boldsymbol{\pi}\cdot\mathbf{B})\theta_3 \quad ,\\
\rho^{(2)} &= e(p_2\theta_1 - p_1\theta_2) \quad ,
\end{aligned}
$$

where p_a are the canonical momenta conjugate to θ_a, and $\rho^{(2)} - \nabla\cdot\boldsymbol{\pi}$ is the constraint. We wish to apply the BFV formalism to this model. The BFV action for the effective Lagrangian density (8) is given by

$$
S_{BFV} = \int d^4x(-\boldsymbol{\pi}\cdot\dot{\mathbf{A}} + \pi_0\dot{\mathrm{A}}_0 + \mathrm{p}_a\dot{\theta}_a + \dot{\mathrm{C}}\mathcal{P} + \dot{\bar{\mathrm{C}}}\bar{\mathcal{P}} - \mathrm{H}_0^{(2)}) + \int \mathrm{dt}[\Psi, \mathrm{Q}]) \quad (9)
$$

Here also π_0 (the conjugate momentum to the scalar potential A_0) has to vanish. We have introduced, keeping the same notation as in the case of electrodynamics, the canonical pair $(C, \mathcal{P})$ corresponding to the constraint $G_1 = \rho^{(2)} - \nabla\cdot\boldsymbol{\pi}$, and the canonical pair $(\bar{C}, \bar{\mathcal{P}})$ corresponding to the constraint $G_2 = \pi_0$. The two constraints are first class. The BRST charge is given by

$$
Q = \int d^3x[C(\rho^{(2)} - \nabla\cdot\boldsymbol{\pi}) + \mathrm{i}\bar{\mathcal{P}}\pi_0] \qquad (10)
$$

and S_{BFV} is invariant under the BRST transformations

$$
\begin{aligned}
s\mathbf{A} &= -\nabla\mathrm{C} \quad , \qquad \mathrm{sC} = 0 \quad ,\\
s\mathcal{P} &= \nabla\cdot\boldsymbol{\pi} - \rho^{(2)} \quad , \qquad s\boldsymbol{\pi} = 0 \quad ,\\
sA_0 &= i\bar{\mathcal{P}} \quad , \qquad s\bar{\mathcal{P}} = 0 \quad ,\\
s\bar{C} &= -i\pi_0 \quad , \qquad s\pi_0 = 0 \quad , \qquad (11)\\
s\theta_1 &= -e\theta_2 C \quad , \qquad s\theta_2 = e\theta_1 C \quad ,\\
sp_1 &= -ep_2 C \quad , \qquad sp_2 = ep_1 C \quad ,\\
s\theta_3 &= 0 \quad , \qquad sp_3 = 0 \quad .
\end{aligned}
$$

As in the previous case we decompose $\mathbf{A}$ and $\boldsymbol{\pi}$ into transverse and longitudinal components and we solve for C, $\bar{\mathcal{P}}$, $\boldsymbol{\pi}^L$ and π_0 using relations from (11)

$$C = -sA^{L'} \quad , \qquad \bar{\mathcal{P}} = -isA_0 \quad , \qquad \boldsymbol{\pi}^L = \frac{\nabla}{\nabla^2}(s\mathcal{P} + \rho^{(2)}) \quad , \qquad \pi_0 = is\bar{C} \quad .$$

After substituting in (9) we end up with an expression for S_{BFV} consisting of an uncanonical part and a BRST exact one as in the case of electrodynamics. The uncanonical part is diagonalized by performing the following Darboux's transformations while the BRST exact one does not change.

$$p_1 \longrightarrow p_1 \cos\alpha + p_2 \sin\alpha \quad , \qquad \theta_1 \longrightarrow \theta_1 \cos\alpha + \theta_2 \sin\alpha \quad ,$$

$$p_2 \longrightarrow p_2 \cos\alpha - p_1 \sin\alpha \quad , \qquad \theta_2 \longrightarrow \theta_2 \cos\alpha - \theta_1 \sin\alpha \quad ,$$

where $\alpha = eA^{L'}$

We have then

$$\begin{aligned}
S_{BFV} &\longrightarrow \int d^4x[-\boldsymbol{\pi}^T \cdot \dot{\mathbf{A}}^{\mathrm{T}} + \mathrm{p}_a\dot{\theta}_a - \mathrm{H}_{\mathrm{C}}^{(2)} + \mathrm{sF}^{(2)}] + \int \mathrm{dt}[\Psi, \mathrm{Q}] \quad , \qquad (12)\\
H_C^{(2)} &= \frac{1}{2}[(\boldsymbol{\pi}^T)^2 + \mathbf{B}^2 - \rho^{(2)}\frac{1}{\nabla^2}\rho^{(2)} + (\nabla\theta_a)^2 + \mathrm{p}_a^2]\\
&\quad + e\mathbf{A}^{\mathrm{T}} \cdot (\theta_1\nabla\theta_2 - \theta_2\nabla\theta_1) + \frac{\mathrm{e}^2}{2}(\mathbf{A}^{\mathrm{T}})^2(\theta_1^2 + \theta_2^2)\\
&\quad - \frac{N_c e^2}{6\pi^2 f_\pi}[\boldsymbol{\pi}^T + \frac{\nabla}{\nabla^2}\rho^{(2)}] \cdot \mathbf{B}\theta_3 \quad ,\\
F^{(2)} &= i\bar{C}\dot{A}_0 + \mathcal{P}\dot{A}^{L'} + \frac{1}{2}(s\mathcal{P})\frac{1}{\nabla^2}\mathcal{P} + \mathcal{P}\frac{1}{\nabla^2}\rho^{(2)} + \frac{N_c e^2}{6\pi^2 f_\pi}(\frac{\nabla}{\nabla^2}\mathcal{P}) \cdot \mathbf{B}\theta_3 \quad ,
\end{aligned}$$

where $H_C^{(2)}$ is the Coulomb gauge Hamiltonian.

Next we take as gauge fermion

$$\Psi = -\int d^3x F^{(2)}$$

and we end up with a Coulomb gauge expression for the effective action with the unphysical $\mathbf{A}^{\mathrm{L}}$ cancelled out. In [6] in the context of the Faddeev-Jackiw formalism we solved the constraint $\rho^{(2)} - \nabla \cdot \boldsymbol{\pi} = 0$ for $\boldsymbol{\pi}^L$ and substituted back into the expression (8) for the effective Lagrangian density. We came

up with an uncanonical expression which was rediagonalized by performing the same Darboux's transformations. The resulting expression for the gauge fixed Lagrangian density is the same as the one obtained in the context of this work.

Replacing the fields by the Darboux transformed ones in (11) we obtain

$$\begin{gathered} sA^{L'} = -\mathrm{C} \;\;, \qquad \mathrm{sC} = 0 \;\;, \\ s\mathcal{P} = \pi^{L'} - \rho^{(2)} \;\;, \qquad s\pi^{L'} = 0 \;\;, \\ sA_0 = i\bar{\mathcal{P}} \;\;, \qquad s\bar{\mathcal{P}} = 0 \;\;, \\ s\bar{C} = -i\pi_0 \;\;, \qquad s\pi_0 = 0 \;\;, \\ s\theta_a = 0 \;\;, \qquad sp_a = 0 \;\;, \qquad a = 1,2,3 \\ s\mathbf{A}^{\mathrm{T}} = 0 \;\;, \qquad \mathrm{s}\boldsymbol{\pi}^{\mathrm{T}} = 0 \;\;, \end{gathered} \tag{13}$$

and

$$\begin{gathered} \sigma(-C) = A^{L'} \;\;, \qquad \sigma A^{L'} = 0 \;\;, \\ \sigma\pi^{L'} = \mathcal{P} \;\;, \qquad \sigma\mathcal{P} = 0 \;\;, \\ \sigma(i\bar{\mathcal{P}}) = A_0 \;\;, \qquad \sigma A_0 = 0 \;\;, \\ \sigma(-i\pi_0) = \bar{C} \;\;, \qquad \sigma(\bar{C}) = 0 \;\;, \\ \sigma\theta_a = 0 \;\;, \qquad \sigma p_a = 0 \;\;, \qquad a = 1,2,3 \\ \sigma\mathbf{A}^{\mathrm{T}} = 0 \;\;, \qquad \sigma\boldsymbol{\pi}^{\mathrm{T}} = 0 \;\;. \end{gathered} \tag{14}$$

From (13) and (14) we conclude that the Darboux transformed fields are σ and s closed (physical). The only fields that are unphysical are actually those that do not appear in the final expression for the gauge fixed action.

3 Keeping third order terms

Now we proceed with expansion and keep terms up to third order in θ_a [6]

$$\mathcal{L}_{eff} = \mathcal{L}_{EM} + \mathcal{L}_{\sigma}^{(2)} + \mathcal{L}_{WZW}^{(2)} + \mathcal{L}_{WZW}^{(3)} + O(\theta^4) \;\;, \tag{15}$$

where the expression for $\mathcal{L}_\sigma^{(2)}$ and $\mathcal{L}_{WZW}^{(2)}$ are given in (7) and

$$
\begin{aligned}
\mathcal{L}_{WZW}^{(3)} = &-\frac{N_c e}{3\pi^2 f_\pi^3}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu)(\theta_1\partial_\alpha\theta_2-\theta_2\partial_\alpha\theta_1)\partial_\beta\theta_3 \\
&+\frac{2N_c e^2}{9\pi^2 f_\pi^3}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu)(\partial_\alpha A_\beta)(\theta_1^2+\theta_2^2)\theta_3 \\
&-\frac{N_c e^2}{3\pi^2 f_\pi^3}\epsilon^{\mu\nu\alpha\beta}A_\mu(\partial_\nu A_\alpha)\theta_3\partial_\beta(\theta_1^2+\theta_2^2) \ .
\end{aligned}
$$

We write (15) as an expression first order in time derivatives

$$
\mathcal{L}_{eff} = -\boldsymbol{\pi}\cdot\dot{\mathbf{A}} + \mathrm{p}_a\dot{\theta}_a - \mathrm{H}_0^{(2)} - \mathrm{H}_0^{(3)} - \mathrm{A}_0(\rho^{(2)}+\rho^{(3)}-\nabla\cdot\boldsymbol{\pi}) + \mathrm{O}(\theta^4) \ , \quad (16)
$$

where the expression for $H_0^{(2)}$ is given in (8) and

$$
\begin{aligned}
H_0^{(3)} = &-\frac{N_c e}{3\pi^2 f_\pi^3}(\boldsymbol{\pi}\times\nabla\theta_3-\mathrm{p}_3\mathbf{B})\cdot(\theta_1\nabla\theta_2-\theta_2\nabla\theta_1) \\
&+\frac{4N_c e^2}{9\pi^2 f_\pi^3}(\boldsymbol{\pi}\cdot\mathbf{B})(\theta_1^2+\theta_2^2)\theta_3 - \frac{\mathrm{N_c e^2}}{3\pi^2\mathrm{f}_\pi^3}(\boldsymbol{\pi}\times\mathbf{A})\cdot[\nabla(\theta_1^2+\theta_2^2)]\theta_3 \\
&-\frac{2N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{A}\cdot\mathbf{B})(\mathrm{p}_1\theta_1+\mathrm{p}_2\theta_2)\theta_3 - \frac{\mathrm{N_c e}}{3\pi^2\mathrm{f}_\pi^3}(\mathbf{B}\cdot\nabla\theta_3)(\mathrm{p}_2\theta_1-\mathrm{p}_1\theta_2) \ , \\
\rho^{(2)} = &\, e(p_2\theta_1-p_1\theta_2) \ , \\
\rho^{(3)} = &-\frac{N_c e^2}{3\pi^2 f_\pi^3}\nabla\cdot[\mathbf{B}(\theta_1^2+\theta_2^2)\theta_3] \ .
\end{aligned}
$$

The BFV effective action is given by

$$
S_{BFV} = \int d^4x(-\boldsymbol{\pi}\cdot\dot{\mathbf{A}}+\pi_0\dot{\mathrm{A}}_0+\mathrm{p}_a\dot{\theta}_a+\dot{\mathrm{C}}\mathcal{P}+\dot{\bar{\mathrm{C}}}\bar{\mathcal{P}}-\mathrm{H}_0^{(2)}-\mathrm{H}_0^{(3)})+\int \mathrm{dt}[\Psi,\mathrm{Q}] \quad (17)
$$

and

$$
Q = \int d^3x[C(\rho^{(2)}+\rho^{(3)}-\nabla\cdot\boldsymbol{\pi})+\mathrm{i}\bar{\mathcal{P}}\pi_0] \quad (18)
$$

is the BRST charge.

S_{BFV} is invariant under the BRST transformations

$$
s\mathbf{A} = -\nabla\mathrm{C} \ , \qquad \mathrm{sC} = 0 \ ,
$$

$$
\begin{aligned}
s\mathcal{P} = \nabla\cdot\boldsymbol{\pi} - \rho^{(2)} - \rho^{(3)} \quad , \qquad s\boldsymbol{\pi} = \frac{N_c e^2}{3\pi^2 f_\pi^3}\nabla[(\theta_1^2+\theta_2^2)\theta_3]\times\nabla C \quad ,\\
sA_0 = i\bar{\mathcal{P}} \quad , \qquad s\bar{\mathcal{P}} = 0 \quad ,\\
s\bar{C} = -i\pi_0 \quad , \qquad s\pi_0 = 0 \quad ,\\
s\theta_1 = -e\theta_2 C \quad , \qquad sp_1 = -ep_2 C - \frac{2N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{B}\cdot\nabla\mathrm{C})\theta_1\theta_3 \quad ,\\
s\theta_2 = e\theta_1 C \quad , \qquad sp_2 = ep_1 C - \frac{2N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{B}\cdot\nabla\mathrm{C})\theta_2\theta_3 \quad ,\\
s\theta_3 = 0 \quad , \qquad sp_3 = -\frac{N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{B}\cdot\nabla\mathrm{C})(\theta_1^2+\theta_2^2) \quad .
\end{aligned}
\tag{19}
$$

We repeate the same procedure as in the previous case by solving for C,$\bar{\mathcal{P}}$,$\boldsymbol{\pi}^L$ and π_0 from (19). The resulting uncanonical term is diagonalized by the following Darboux's transformations.

$$
\begin{aligned}
&\theta_1 \to \theta_1\cos\alpha + \theta_2\sin\alpha \quad ,\\
&\theta_2 \to \theta_2\cos\alpha - \theta_1\sin\alpha \quad ,\\
&\boldsymbol{\pi}^T \to \boldsymbol{\pi}^T - \tfrac{N_c e^2}{3\pi^2 f_\pi^3}\nabla[(\theta_1^2+\theta_2^2)\theta_3]\times\mathbf{A}^{\mathrm{L}} \quad ,\\
&p_1 \to p_1\cos\alpha + p_2\sin\alpha + \tfrac{2N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{B}\cdot\mathbf{A}^{\mathrm{L}})(\theta_1\cos\alpha+\theta_2\sin\alpha)\theta_3 \quad ,\\
&p_2 \to p_2\cos\alpha - p_1\sin\alpha + \tfrac{2N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{B}\cdot\mathbf{A}^{\mathrm{L}})(\theta_2\cos\alpha-\theta_1\sin\alpha)\theta_3 \quad ,\\
&p_3 \to p_3 + \tfrac{N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{B}\cdot\mathbf{A}^{\mathrm{L}})(\theta_1^2+\theta_2^2) \quad .
\end{aligned}
$$

We have then

$$
S_{BFV} \to \int d^4x[-\boldsymbol{\pi}^T\cdot\dot{\mathbf{A}}^{\mathrm{T}} + \mathrm{p}_a\dot{\theta}_a - \mathrm{H}_{\mathrm{C}}^{(2)} - \mathrm{H}_{\mathrm{C}}^{(3)} + \mathrm{s}(\mathrm{F}^{(2)}+\mathrm{F}^{(3)})] + \int \mathrm{dt}[\Psi,\mathrm{Q}] \tag{20}
$$

where $H_C^{(2)}$ and $F^{(2)}$ are given in (12),

$$
\begin{aligned}
H_C^{(3)} = &-\frac{N_c e}{3\pi^2 f_\pi^3}(\boldsymbol{\pi}^T\times\nabla\theta_3 - \mathrm{p}_3\mathbf{B})\cdot(\theta_1\nabla\theta_2 - \theta_2\nabla\theta_1)\\
&+\frac{4N_c e^2}{9\pi^2 f_\pi^3}(\boldsymbol{\pi}^T\cdot\mathbf{B})(\theta_1^2+\theta_2^2)\theta_3 - \frac{\mathrm{N_c e^2}}{3\pi^2\mathrm{f}_\pi^3}(\boldsymbol{\pi}^{\mathrm{T}}\times\mathbf{A}^{\mathrm{T}})\cdot[\nabla(\theta_1^2+\theta_2^2)]\theta_3\\
&-\frac{2N_c e^2}{3\pi^2 f_\pi^3}(\mathbf{A}^{\mathrm{T}}\cdot\mathbf{B})(\mathrm{p}_1\theta_1+\mathrm{p}_2\theta_2)\theta_3 - \frac{\mathrm{N_c e}}{3\pi^2\mathrm{f}_\pi^3}(\mathbf{B}\cdot\nabla\theta_3)(\mathrm{p}_2\theta_1 - \mathrm{p}_1\theta_2) \quad ,
\end{aligned}
$$

and

$$F^{(3)} = \mathcal{P}\frac{1}{\nabla^2}\rho^{(3)} + \frac{N_c e}{3\pi^2 f_\pi^3}[(\frac{\nabla}{\nabla^2}\mathcal{P}) \times \nabla\theta_3] \cdot (\theta_1\nabla\theta_2 - \theta_2\nabla\theta_1)$$
$$- \frac{4N_c e^2}{9\pi^2 f_\pi^3}(\frac{\nabla}{\nabla^2}\mathcal{P}) \cdot \mathbf{B}(\theta_1^2 + \theta_2^2)\theta_3 + \frac{\mathrm{N_c e^2}}{3\pi^2 \mathrm{f}_\pi^3}[(\frac{\nabla}{\nabla^2}\mathcal{P}) \times \mathbf{A}_\mathrm{T}] \cdot [\nabla(\theta_1^2 + \theta_2^2)]\theta_3 \ \ .$$

Then by fixing the gauge fermion

$$\Psi = -\int d^3x(F^{(2)} + F^{(3)})$$

we end up with a Coulomb gauge expression for the effective action as in the previous case. The same result was obtained in [6] using the Faddeev-Jackiw method.

If we perform the Darboux' transformations in (19) we get

$$\begin{gathered}
sA^{L'} = -\mathrm{C} \ \ , \qquad \mathrm{sC} = 0 \ \ , \\
s\mathcal{P} = \pi^{L'} - \rho^{(2)} - \rho^{(3)} \ \ , \qquad s\pi^{L'} = 0 \ \ , \\
sA_0 = i\bar{\mathcal{P}} \ \ , \qquad s\bar{\mathcal{P}} = 0 \ \ , \\
s\bar{C} = -i\pi_0 \ \ , \qquad s\pi_0 = 0 \ \ , \\
s\theta_a = 0 \ \ , \qquad sp_a = 0 \ \ , \qquad a = 1,2,3 \\
s\mathbf{A}^\mathrm{T} = 0 \ \ , \qquad \mathrm{s}\boldsymbol{\pi}^\mathrm{T} = 0 \ \ ,
\end{gathered} \tag{21}$$

and

$$\begin{gathered}
\sigma(-C) = A^{L'} \ \ , \qquad \sigma A^{L'} = 0 \ \ , \\
\sigma\pi^{L'} = \mathcal{P} \ \ , \qquad \sigma\mathcal{P} = 0 \ \ , \\
\sigma(i\bar{\mathcal{P}}) = A_0 \ \ , \qquad \sigma A_0 = 0 \ \ , \\
\sigma(-i\pi_0) = \bar{C} \ \ , \qquad \sigma(\bar{C}) = 0 \ \ , \\
\sigma\theta_a = 0 \ \ , \qquad \sigma p_a = 0 \ \ , \qquad a = 1,2,3 \\
\sigma\mathbf{A}^\mathrm{T} = 0 \ \ , \qquad \sigma\boldsymbol{\pi}^\mathrm{T} = 0 \ \ .
\end{gathered} \tag{22}$$

Here again we see that only the fields that appear in the gauge fixed expression for the effective action are s and σ closed (physical).

4 Conclusion

In this work we drew the analogy between the BFV formalism and the Faddeev-Jackiw approach in two cases. The spinor electrodynamics and the 4-dimensional $U_{EM}(1)$ gauged SU(2) WZW model. According to the BFV formalism the scalar potential A_0 was promoted to a full dynamical variable with vanishing conjugate momentum π_0, and the phase space was extended by introducing a ghost field for every constraint. The BFV action was written as a sum of an uncanonical term and a BRST exact one. Darboux's transformations were used to diagonalize the uncanonical term and the gauge fermion were chosen to cancel the BRST exact one. The resulting expression for the Coulomb gauge effective action in both cases are the same as the ones obtaind by the Faddeev-Jackiw method [6]. We also showed that the Darboux transformed fields are BRST and σ closed.

The SU(3) case is currently under investigation.

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5 Appendix

Our metric is $g_{\mu\nu} = diag(1,-1,-1,-1)$. We choose $e > 0$. We define $\epsilon^{0123} = 1$. By ρ we denote the spinor charge density $e\psi^\dagger\psi$. By $\boldsymbol{\pi}$ we denote the electric field $\mathbf{E}$ so that (π_μ, A^μ) $\mu = 0,1,2,3$ is a canonical pair. We made use of the following Poisson brackets

$$\begin{aligned}
[A^\mu(\mathbf{x},\mathrm{t}), \pi^\nu(\mathbf{y},\mathrm{t})] &= g^{\mu\nu}\delta(\mathbf{x}-\mathbf{y}) \ , \\
[\theta_a(\mathbf{x},\mathrm{t}), \mathrm{p}_b(\mathbf{y},\mathrm{t})] &= \delta_{ab}\delta(\mathbf{x}-\mathbf{y}) \ , \\
[\psi_a(\mathbf{x},\mathrm{t}), \psi_b^\dagger(\mathbf{y},\mathrm{t})] &= -i\delta_{ab}\delta(\mathbf{x}-\mathbf{y}) \ , \\
[C(\mathbf{x},\mathrm{t}), \mathcal{P}(\mathbf{y},\mathrm{t})] &= -\delta(\mathbf{x}-\mathbf{y}) \ , \\
[\bar{C}(\mathbf{x},\mathrm{t}), \bar{\mathcal{P}}(\mathbf{y},\mathrm{t})] &= -\delta(\mathbf{x}-\mathbf{y}) \ .
\end{aligned}$$

The Grassmann parities of the fields are given by $\epsilon_{A_\mu} = \epsilon_{\pi_\mu} = \epsilon_{\theta_a} = \epsilon_{p_a} = 0$, $\epsilon_\psi = \epsilon_{\psi^\dagger} = \epsilon_C = \epsilon_{\mathcal{P}} = \epsilon_{\bar{C}} = \epsilon_{\bar{\mathcal{P}}} = 1$ and their ghost number $gh(C) = -gh(\mathcal{P}) = 1$, $gh(\bar{C}) = -gh(\bar{\mathcal{P}}) = -1$, $gh(A_\mu) = gh(\pi_\mu) = gh(\theta_a) = gh(p_a) = 0$.

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